







Elasticity Tensors

- ◆ Tensor is a 3D representation of some physical state
 - Stress tensor **T**
 - Strain tensor ϵ
 - Elasticity tensor **C** (Compliance tensor **S**)

Stress and strain

- ◆ $\mathbf{T} = \mathbf{C} \epsilon$
- ◆ For an anisotropic material, C has 36 independent components
 - C_{ij} for $i, j = 1-6$
 - Anisotropic: properties depend on direction of measurement

Elastic Constants

- ◆ Young's modulus (E)
- ◆ Poisson's ratio ()
- ◆ Bulk modulus (K)
- ◆ Shear modulus (G)
- ◆ Lamé's constant ()
- ◆ For an isotropic material, elastic constants are CONSTANT

Elastic Constants

- ◆ Elastic constants have directional dependence for non-isotropic materials
- ◆ How do the elastic constants relate to the elasticity matrix:
 - For an orthotropic material:
 - ◆ $C_{11} = E_1$
 - ◆ $C_{12} = f(E_1, E_2, \text{Poisson's ratio})$
 - ◆ C_{44} to $C_{66} = G_{12}/2$ etc.

Elastic Constants

- ◆ Young's modulus:
 - Relationship between tensile or compressive stress and strain
 - Applies for small strains (within the elastic range)
 - $\sigma = E \epsilon$

Elastic Constant

- ◆ Poisson's ratio
 - Describes lateral deformation in response to an axial load
 - $\nu = - \frac{\epsilon_{\text{lat}}}{\epsilon_{\text{axial}}}$
- ◆ Shear modulus
 - Describes relationship between applied torque and angle of deformation
 - $G = M_t/I_p$
 - $G = \frac{E}{2(1+\nu)}$
 - ◆ G_{12} vs. G_{21}
 - ◆ $G_{12} = 1/2(G_{21})$

Elastic Constant

- ◆ Bulk modulus
 - Describes the change in volume in response to hydrostatic pressure (equal stresses in all directions)
 - $K = -P/e$ ($e = \Delta \text{vol}/\text{vol}$)
- ◆ Lamé's constant
 - Fifth elastic constant

Elastic Constants

- ◆ Isotropic materials:
 - 2 independent elastic constants
 - Define E, ν :
 - ◆ $E = E$
 - ◆ $\nu = \nu$
 - ◆ $\nu = E/[2(1 + \nu)]$
 - ◆ $G = E/[2(1 + \nu)]$
 - ◆ $K = E/[3(1 - 2\nu)]$

Tensors

- ◆ Homogeneous behavior
 - Material properties are independent of location within a material
 - C_{ijkl} are independent of x, y, z

Stress-strain relationships

- ◆ $\sigma_{11} = C_{11} \epsilon_{11}$
 - $\sigma_{11} = E_1 \epsilon_{11}$
- ◆ $\sigma_{12} = C_{12} \epsilon_{12}$
 - $\sigma_{12} = 2G \epsilon_{12} = G \gamma_{12}$
- ◆ $\epsilon_{11} = 1/E [\sigma_{11} - \nu (\sigma_{22} + \sigma_{33})]$
- ◆ $\epsilon_{12} = \gamma_{12}/2G$

Material Models

- ◆ Anisotropy
 - 36 independent components elasticity matrix
- ◆ Orthotropy
 - 9 independent components to elasticity matrix
- ◆ Transverse isotropy
 - 3 independent components
- ◆ Isotropy
 - 2 independent components
