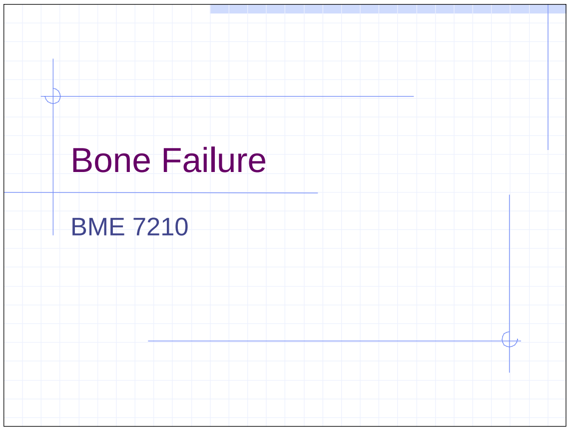

Bone Failure

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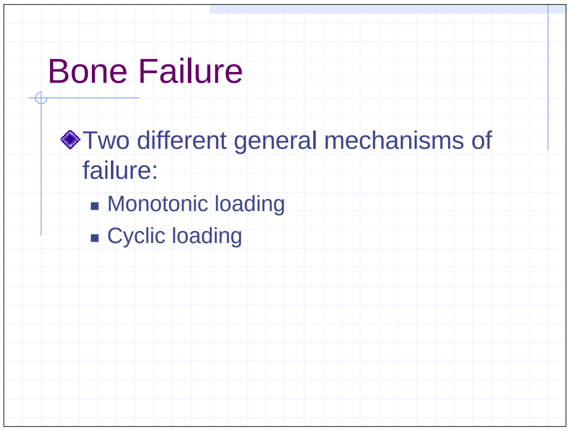
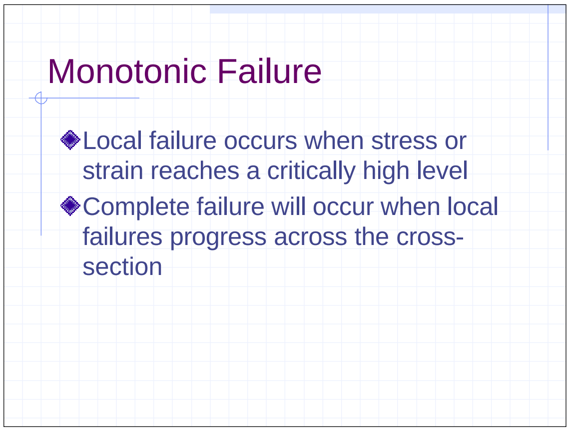


Bone Failure

- Two different general mechanisms of failure:
  - Monotonic loading
  - Cyclic loading



Monotonic Failure

- Local failure occurs when stress or strain reaches a critically high level
- Complete failure will occur when local failures progress across the cross-section

Monotonic Failure Criteria

- Normal stresses:
  - $\sigma_{zz}^{ult} = F_{zz}^{ult}/A$
- Looking at ultimate normal stress as a function of apparent density (volume fraction)
  - $\sigma_{zz}^{ult} = C\rho^2$
  - Applies to cortical and trabecular bone

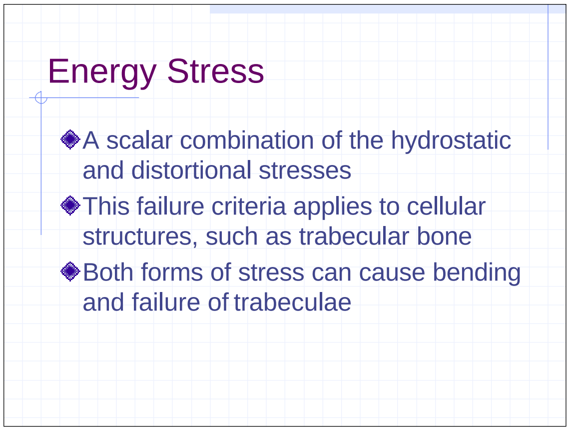
Monotonic, Multiaxial Failure

- Multiaxial stresses result in multiple normal and shear stresses within a specimen
- May cause failure to occur when σ_{zz} is less than σ_{zz}^{ult}
- Must consider other failure criteria in this case

Multiaxial Failure Criteria

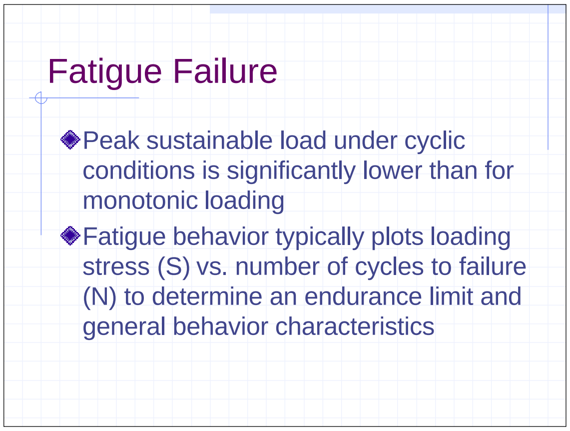
- Maximum stress criterion:
  - Failure occurs when any principal stress component exceeds σ_{zz}^{ult}
- Energy stress failure criterion:
  - Material failure occurs when the local value of the energy stress reaches a critical value

$$\bar{\sigma} = \bar{\sigma}^{ult} = \sigma_{zz}^{ult}$$



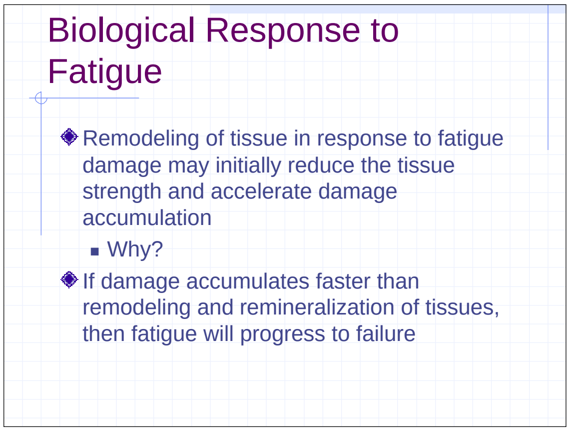
Energy Stress

- A scalar combination of the hydrostatic and distortional stresses
- This failure criteria applies to cellular structures, such as trabecular bone
- Both forms of stress can cause bending and failure of trabeculae



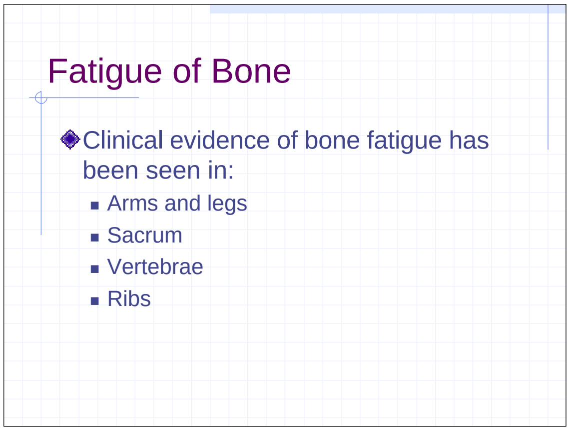
Fatigue Failure

- Peak sustainable load under cyclic conditions is significantly lower than for monotonic loading
- Fatigue behavior typically plots loading stress (S) vs. number of cycles to failure (N) to determine an endurance limit and general behavior characteristics



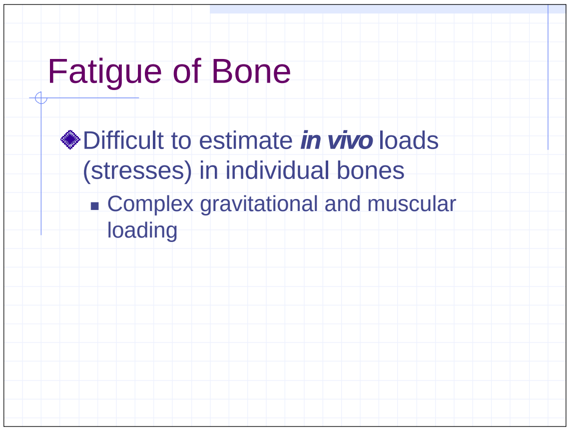
Biological Response to Fatigue

- Remodeling of tissue in response to fatigue damage may initially reduce the tissue strength and accelerate damage accumulation
  - Why?
- If damage accumulates faster than remodeling and remineralization of tissues, then fatigue will progress to failure



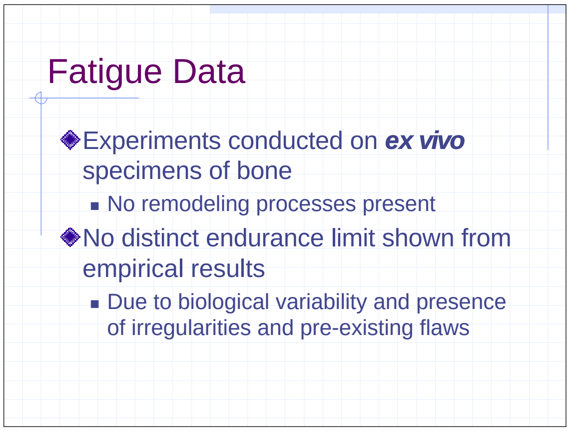
Fatigue of Bone

- Clinical evidence of bone fatigue has been seen in:
  - Arms and legs
  - Sacrum
  - Vertebrae
  - Ribs



Fatigue of Bone

- Difficult to estimate ***in vivo*** loads (stresses) in individual bones
  - Complex gravitational and muscular loading



Fatigue Data

- Experiments conducted on ***ex vivo*** specimens of bone
  - No remodeling processes present
- No distinct endurance limit shown from empirical results
  - Due to biological variability and presence of irregularities and pre-existing flaws

Fatigue Data

- Appears to be a “critical strain” level above which the damage accumulates quickly
  - Tensile strains: 2500 με
  - Compressive strains: 4000 με
- Critical strains appear to occur at approximately 30% of the ultimate strength in each mode of loading
- Strain range is a better predictor of fatigue life than stress range
  - Change in modulus does not affect yield strain but does affect yield stress

Fatigue Data

- Fatigue life of bone is a function of:
  - Loading mode (tensile vs. compressive)
  - Frequency
  - Temperature*
  - Microstructure
  - Density

Fatigue Data - Cortical Bone

- Fatigue life (N) decreases as:
  - Bone is remodeled from primary osteonal to secondary osteonal
  - Interstitial bone fraction increases with respect to number of complete osteons
  - Bone is loaded in tension instead of compression
- Why?

Fatigue Data

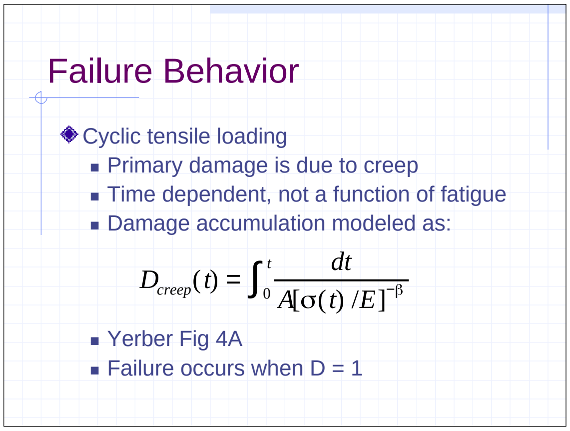
- Cycles to failure for uniaxially loaded specimens tend to be lower than for specimens tested in bending
  - Bending causes high strains only in outer regions of specimen
  - Greater loaded area means higher chance of voids or pre-existing cracks that will reduce the strength of the specimen

Fatigue Data - Trabecular Bone

- Less studied than cortical bone
- Cortical material compared to trabecular material showed that cortical bone had higher fatigue strength than trabeculae
  - Different microstructure
- Trabecular tissue specimens show increasing hysteresis throughout cyclic loading
- Trabecular fatigue behavior thought to be generally similar to that of cortical bone

Failure Behavior

- Under sustained cyclic loads that cycle around a non-zero level, both fatigue damage (due to cycling) and creep damage (due to non-zero mean load) will occur
- These phenomena appear to interact to accelerate failure

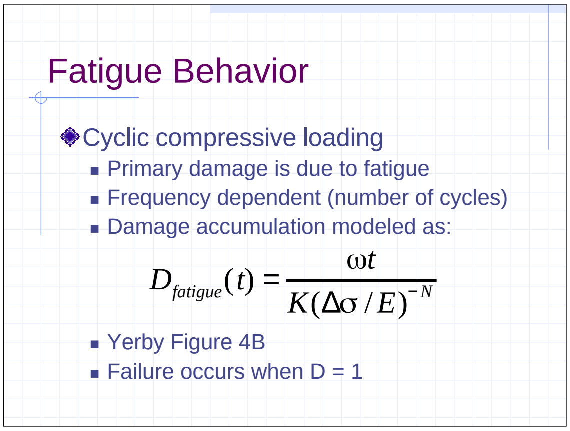


Failure Behavior

- Cyclic tensile loading
  - Primary damage is due to creep
  - Time dependent, not a function of fatigue
  - Damage accumulation modeled as:

$$D_{creep}(t) = \int_0^t \frac{dt}{A[\sigma(t)/E]^{-\beta}}$$

  - Yerber Fig 4A
  - Failure occurs when D = 1

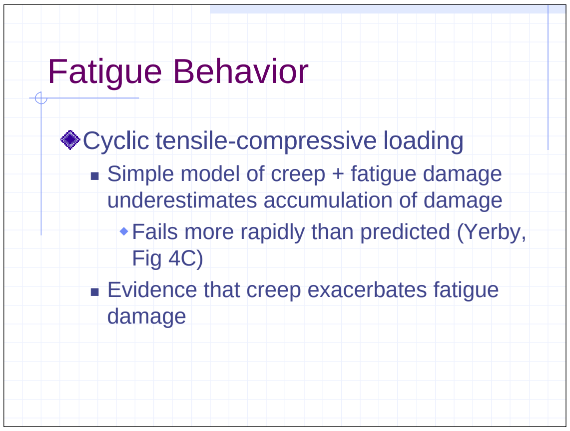


Fatigue Behavior

- Cyclic compressive loading
  - Primary damage is due to fatigue
  - Frequency dependent (number of cycles)
  - Damage accumulation modeled as:

$$D_{fatigue}(t) = \frac{\omega t}{K(\Delta\sigma / E)^{-N}}$$

  - Yerby Figure 4B
  - Failure occurs when D = 1



Fatigue Behavior

- Cyclic tensile-compressive loading
  - Simple model of creep + fatigue damage underestimates accumulation of damage
    - Fails more rapidly than predicted (Yerby, Fig 4C)
  - Evidence that creep exacerbates fatigue damage

Fatigue Behavior - Multiaxial

- All stress tensor values may be non-zero
- Multiaxial fatigue estimate for bone

$$N = A\left(\frac{\Delta\bar{\sigma}}{\bar{\sigma}^{ult}}\right)^{b}$$

$$\bar{\sigma}^{ult} = C\rho^{2}$$

Fatigue and Energy Loss

- Energy transferred to the material during a cycle of loading is indicated by the hysteresis loop
- Bone exhibits a conditioning effect
  - During intial cycles, the hysteresis loop is reduced in size and stablizes
  - Additional cycles cause a very gradual increase in energy transfer throughout the fatigue life

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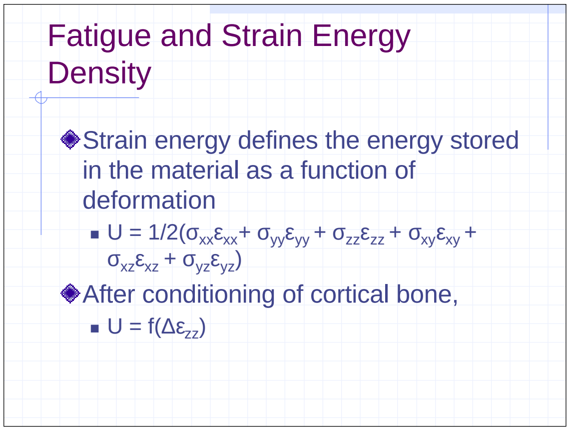
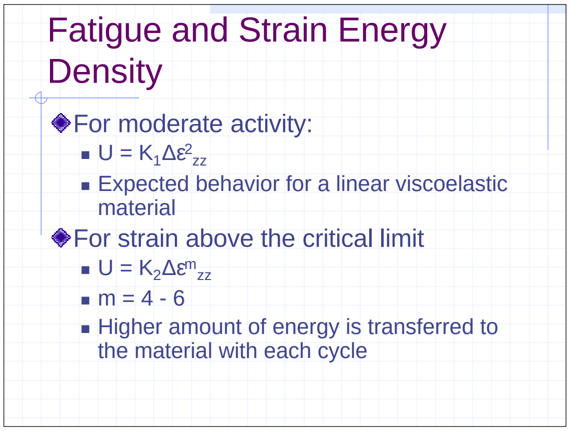
Fatigue and Energy Loss

- The gradual increase in energy transfer (size of hysteresis loop) is accompanied by a decrease in tissue elastic modulus and an increase in damage accumulation
  - Damage can be estimated by:
    - $D = 1 - (E/E_O)$
- These changes occur rapidly towards the end of the fatigue life



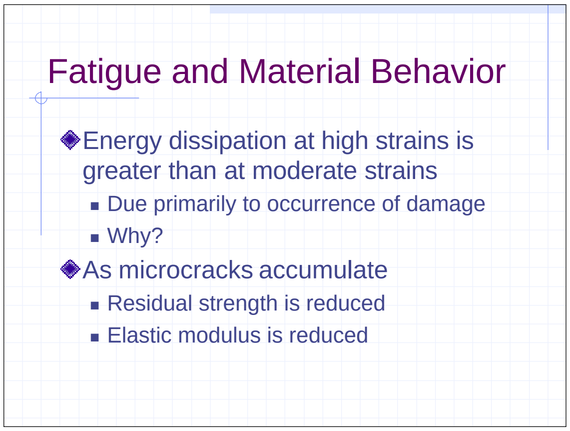
Fatigue and Strain Energy Density

- Strain energy defines the energy stored in the material as a function of deformation
  - $U = 1/2(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \sigma_{xy}\varepsilon_{xy} + \sigma_{xz}\varepsilon_{xz} + \sigma_{yz}\varepsilon_{yz})$
- After conditioning of cortical bone,
  - $U = f(\Delta\varepsilon_{zz})$



Fatigue and Strain Energy Density

- For moderate activity:
  - $U = K_1\Delta\varepsilon^2_{zz}$
  - Expected behavior for a linear viscoelastic material
- For strain above the critical limit
  - $U = K_2\Delta\varepsilon^m_{zz}$
  - $m = 4 - 6$
  - Higher amount of energy is transferred to the material with each cycle



Fatigue and Material Behavior

- Energy dissipation at high strains is greater than at moderate strains
  - Due primarily to occurrence of damage
  - Why?
- As microcracks accumulate
  - Residual strength is reduced
  - Elastic modulus is reduced

Safety Factors

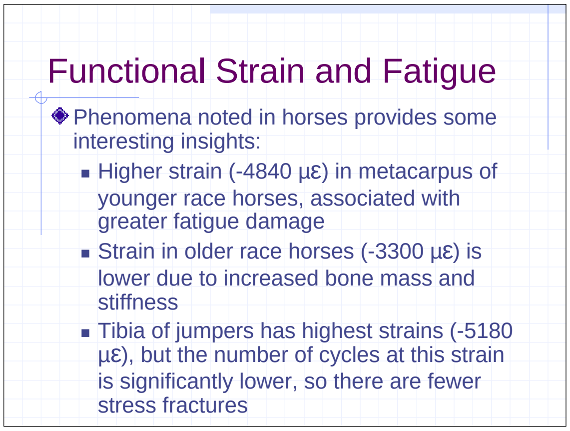
- Defined as the ratio of a structure's failure strength to the maximum allowable, expected stress
- For bone, safety factors will depend on:
  - ***In vivo***, physiological stresses
  - Variation in bone strength
- For a nominally linear, elastic material, safety factors can be examined in terms of strains as well

Functional Strain

- Vary based on:
  - Anatomical location
  - Functional activity
  - Region within a bone
- Peak strains have been experimentally measured on the outer cortex of some animals and humans
  - Range from -1700 to -5200 με in distal limbs during strenuous activities

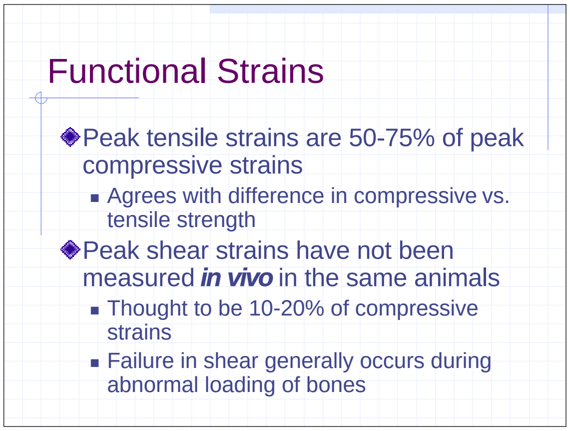
Functional Strains

- Most principal strains are oriented along the longitudinal axis of the bone
  - Exceptions: humerus of birds and bats, tibia of sheep
    - Indicates torsional loading
- Range of values due to:
  - Differences in intensity of activities
  - Experimental differences
  - Biological variation



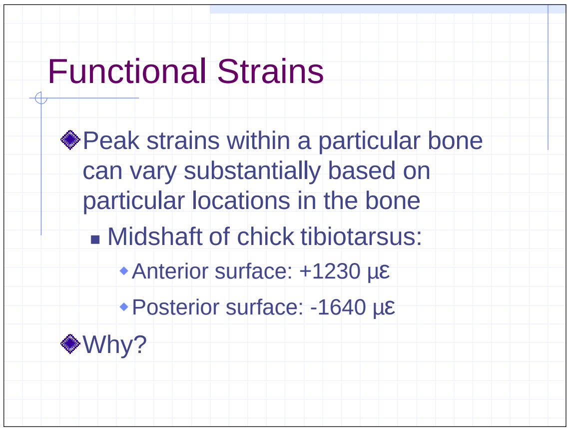
Functional Strain and Fatigue

- Phenomena noted in horses provides some interesting insights:
  - Higher strain (-4840 με) in metacarpus of younger race horses, associated with greater fatigue damage
  - Strain in older race horses (-3300 με) is lower due to increased bone mass and stiffness
  - Tibia of jumpers has highest strains (-5180 με), but the number of cycles at this strain is significantly lower, so there are fewer stress fractures



Functional Strains

- Peak tensile strains are 50-75% of peak compressive strains
  - Agrees with difference in compressive vs. tensile strength
- Peak shear strains have not been measured ***in vivo*** in the same animals
  - Thought to be 10-20% of compressive strains
  - Failure in shear generally occurs during abnormal loading of bones



Functional Strains

- Peak strains within a particular bone can vary substantially based on particular locations in the bone
  - Midshaft of chick tibiotarsus:
    - Anterior surface: +1230 με
    - Posterior surface: -1640 με
- Why?

Functional Strains

- Internal functional strains can only be estimated from finite element models
    - Requires accurate knowledge of functional loading
- Known that a bone loaded in bending will have lower functional strains towards the neutral axis
    - Safety factors will not be constant through a bone, but will vary based on cross-sectional and longitudinal location

Bone Failure Properties

- Cortical bone
    - Ultimate strain: -14000 to -21000 με
    - Yield strain: -6000 to -8000 με
- Skeletal safety factors:
    - Ultimate strain: 3.4 to 10.3
    - Yield strain: 1.4 to 4.1

Ultimate vs. Yield Strain

- Which safety factor is more appropriate to examine in terms of “failure”?

Safety Factor and Age

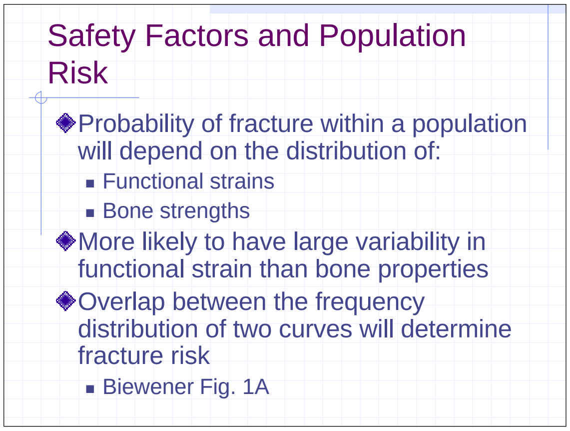
- Bone mineralization increases with age
  - Increases strength through age 40
  - Further increase in mineralization reduces ability to absorb energy
- Safety factor of bone declines with age
  - But maximum functional load can also be expected to decline with age
  - Why?

Personal Fracture Risk

- Probability of fracture depends on:
  - Bone strength in relation to peak functional strain
  - Variability and frequency of loading events
    - Strain history
  - Variation in quality of bone over time

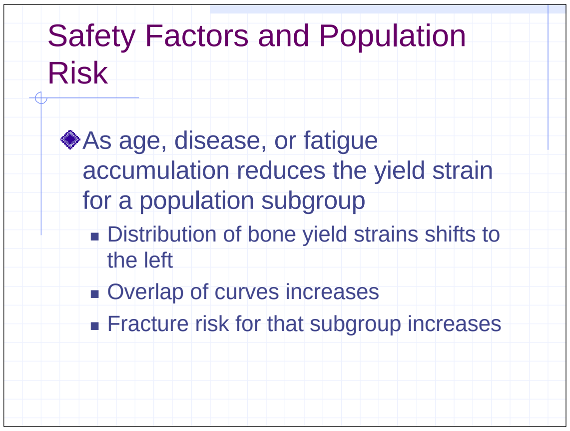
Biological Cost

- The physiological effect of a fracture will determine its biological cost
  - Ribs vs. femur
- Bones for which fracture will have a higher biological cost would be expected to have a higher factor of safety
  - Balanced with physiological cost of overly strong bones



Safety Factors and Population Risk

- Probability of fracture within a population will depend on the distribution of:
  - Functional strains
  - Bone strengths
- More likely to have large variability in functional strain than bone properties
- Overlap between the frequency distribution of two curves will determine fracture risk
  - Biewener Fig. 1A



Safety Factors and Population Risk

- As age, disease, or fatigue accumulation reduces the yield strain for a population subgroup
  - Distribution of bone yield strains shifts to the left
  - Overlap of curves increases
  - Fracture risk for that subgroup increases