Musculoskeletal Dynamics

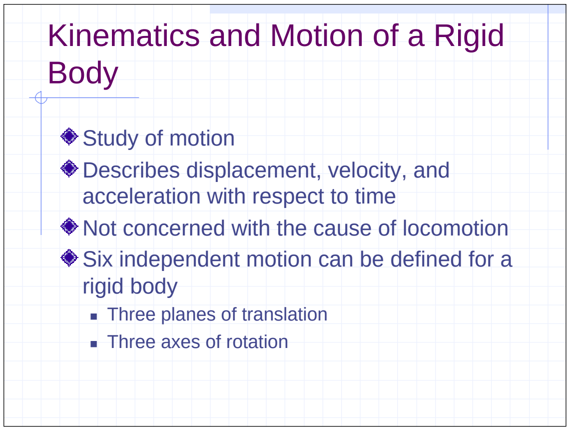
BME 5210 -
Musculoskeletal Biomechancs

Objectives

- Be able to determine movement of body segments based on kinematic data
  - Calculate segment and joint angles
  - Calculate segment velocity and acceleration
- Be able to calculate joint forces from kinematic and kinetic data
- Be able to estimate muscle forces from equilibrium equations
- Discuss how joint moments and muscle forces vary during activities of daily living

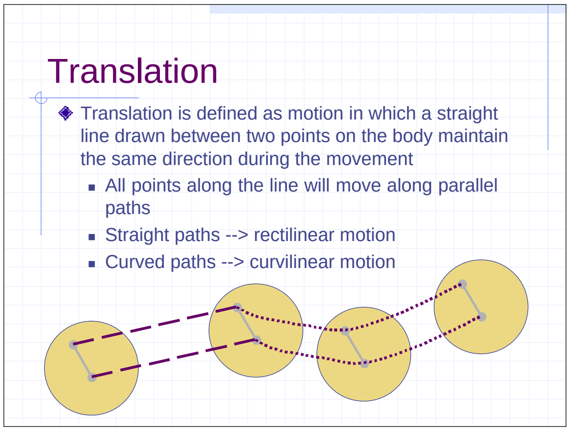
Background Concepts

- Linear and angular displacement
- Velocity
- Acceleration
- Newton's second law
- Anatomical Concepts
  - Body segments:
    - Thigh - leg - foot
    - Arm - forearm - hand



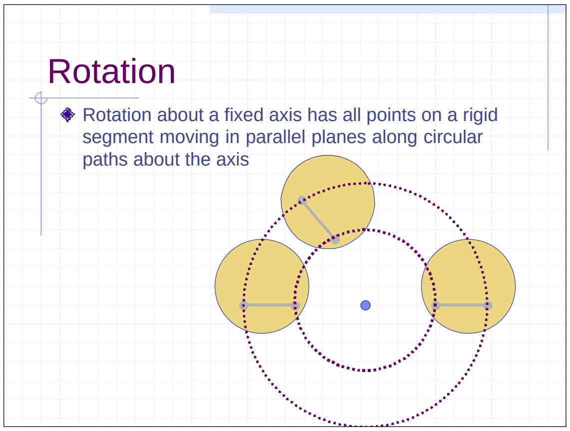
Kinematics and Motion of a Rigid Body

- Study of motion
- Describes displacement, velocity, and acceleration with respect to time
- Not concerned with the cause of locomotion
- Six independent motion can be defined for a rigid body
  - Three planes of translation
  - Three axes of rotation



Translation

- Translation is defined as motion in which a straight line drawn between two points on the body maintain the same direction during the movement
  - All points along the line will move along parallel paths
  - Straight paths --> rectilinear motion
  - Curved paths --> curvilinear motion



Rotation

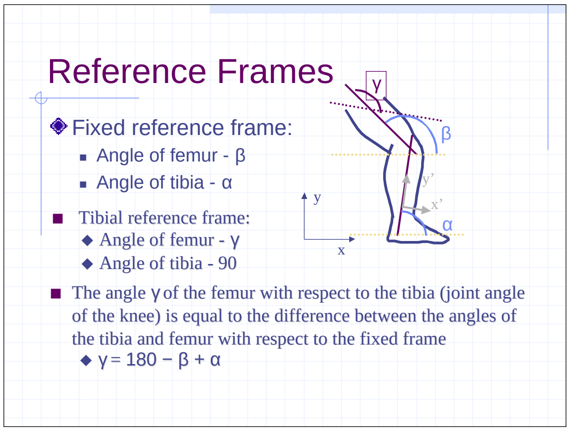
- Rotation about a fixed axis has all points on a rigid segment moving in parallel planes along circular paths about the axis

Motion of a Rigid Body

- All motions can be decomposed into the three translations and three rotations
- For a motion to be characterized, the following must be defined:
  - Type of motion (linear or angular)
  - The reference frame
  - Degrees of freedom (2D or 3D motion)
- Movement of a joint can be described in terms of the relative displacements (linear or angular) of one segment to the adjacent, connected segment

Reference Frames

- Reference frames can be either fixed or moving
- The analysis of motion will be the same, irrespective of the selected reference frame, but the components of the motion will be different
- Optical measurements typically measure movement with respect to a fixed, laboratory reference frame
- Goniometric measurements measure with respect to a moving reference frame
  - One end of goniometer is fixed to distal segment
  - Opposite end is attached to the proximal segment

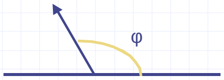


Reference Frames

- Fixed reference frame:
  - Angle of femur - β
  - Angle of tibia - α
- Tibial reference frame:
  - Angle of femur - γ
  - Angle of tibia - 90
- The angle γ of the femur with respect to the tibia (joint angle of the knee) is equal to the difference between the angles of the tibia and femur with respect to the fixed frame
  - $\gamma = 180 - \beta + \alpha$

Relative Segmental Angles

- For gait, relative angle measurements are reported for the hip, knee, and ankle
- During normal gait, the time course of the relative angle measurements are quite reproducible
- Peak amplitudes of the angle measurements are a function of walking speed
- Kinematic Convention: angles are measured in the counterclockwise direction from a 0° horizontal

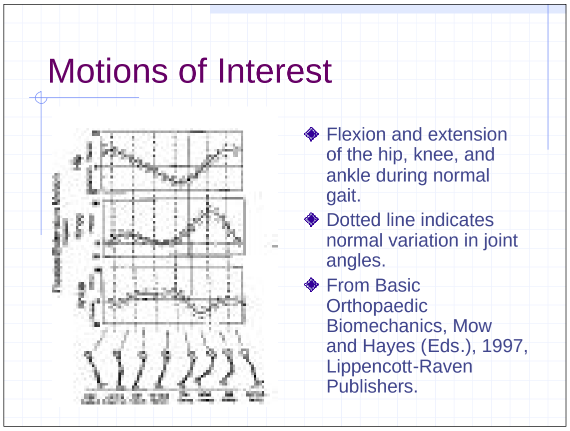


Motions of Interest

- Sagittal, angular motion of the hip, knee, and ankle
- Out-of plane motions of the tibiofemoral joint
  - Abduction-adduction rotation
  - Internal-external rotation
  - Medial-lateral translation (shift)
  - Anterior-posterior translation (draw)
  - Superior-inferior translation (distraction-compression)
- Other motions:
  - Abduction-adduction rotation at the hip and ankle
  - Internal-external rotation of thigh and foot

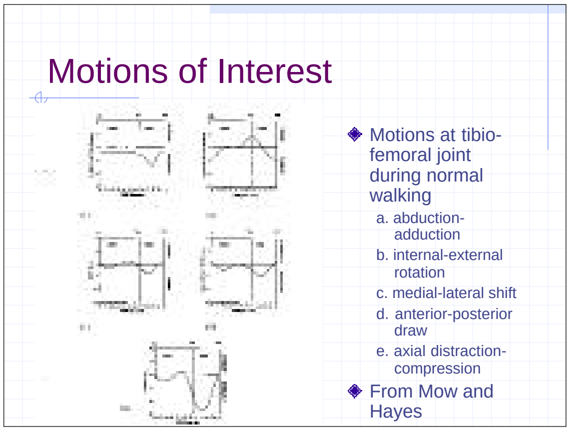
Motions of Interest

- Gait is divided into eight phases
  - 1. Initial contact
  - 2. Loading response
  - 3. Mid-stance
  - 4. Terminal stance
  - 5. Pre-swing
  - 6. Initial swing
  - 7. Mid-swing
  - 8. Terminal swing



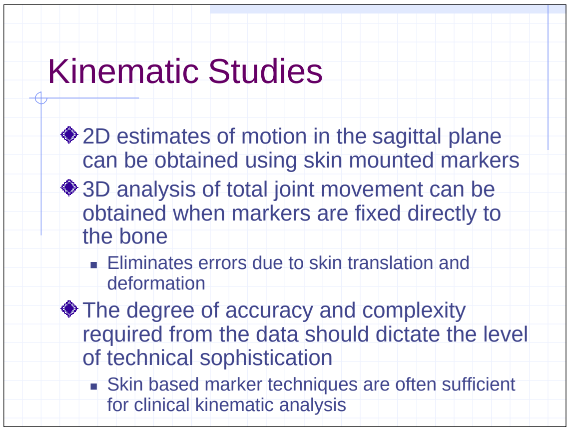
Motions of Interest

- Flexion and extension of the hip, knee, and ankle during normal gait.
- Dotted line indicates normal variation in joint angles.
- From Basic Orthopaedic Biomechanics, Mow and Hayes (Eds.), 1997, Lippencott-Raven Publishers.



Motions of Interest

- Motions at tibio-femoral joint during normal walking
  - a. abduction-adduction
  - b. internal-external rotation
  - c. medial-lateral shift
  - d. anterior-posterior draw
  - e. axial distraction-compression
- From Mow and Hayes

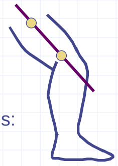


Kinematic Studies

- 2D estimates of motion in the sagittal plane can be obtained using skin mounted markers
- 3D analysis of total joint movement can be obtained when markers are fixed directly to the bone
  - Eliminates errors due to skin translation and deformation
- The degree of accuracy and complexity required from the data should dictate the level of technical sophistication
  - Skin based marker techniques are often sufficient for clinical kinematic analysis

Kinematic Studies

- Segment angles:
  - Calculated based on the position of two segment markers placed along long axis of segment in plane of angle
  - $\Phi_{ij} = \arctan [(y_j - y_i)/(x_j - x_i)]$
- Segment linear velocities and accelerations:
  - Use two data intervals to minimize error
  - $v_{x(i)} = (x_{i+1} - x_{i-1})/(2\Delta t)$
  - $A_{x(i)} = [v_{x(i+1)} - v_{x(i-1)}]/(2\Delta t)$
- Segment angular velocities and accelerations:
  - $\omega_{x(i)} = [\varphi_{(i+1)} - \varphi_{(i-1)}]/(2\Delta t)$
  - $\alpha_{x(i)} = [\omega_{(i+1)} - \omega_{(i-1)}]/(2\Delta t)$



Kinematic Studies

- Kinematic Convention:
  - Angles are positive when directed counterclockwise
  - Velocities are positive when the coordinate component is increasing
  - Accelerations are positive when the velocity component is increasing

Kinematic Studies

- Example: (right leg)
  - $\omega = -2.34$ rad/s $\alpha = 14.29$ rad/s^2
    - Leg is rotating clockwise (negative angular velocity)
    - Leg is decelerating (direction of α is opposite of ω)
  - $v_x = 0.783$ m/s $a_x = -9.27$ m/s^2
  - $v_y = 0.021$ m/s $a_y = -0.31$ m/s^2
    - Center of mass of leg is moving forward and slightly upward (positive velocities)
    - Leg is decelerating (direction of a is opposite of v)
      - Given small initial v_y, the leg will soon be accelerating downwards

Kinetics

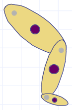
- Kinematics provides information on limb motion, but not the joint forces and moments
- Kinetics assesses the balance of external and internal forces across joints
- Based on Newton's second law
  - F = ma
  - "If the resultant force acting on a body is not zero, the body will have an acceleration proportional to the magnitude and in the direction of this resultant force."

Kinetics - General Approach

- Assumptions:
  - Each body segment is symmetric about its principal axis
  - Angular velocity and acceleration around the longitudinal axis of the segment are negligible
- Steps:
  - 1. Establishment of a link model
    - Inertial properties are lumped at the center of mass of each segment
  - 2. Measurement of necessary data
    - Ground reaction forces
    - Limb segment inertial properties
    - Location of 3D position of joint centers in space and time

Analysis of Kinetics

- Link model:

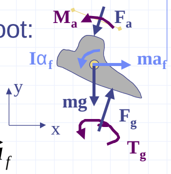


- Free-body diagram including:
  - Intersegmental forces and moments acting at joint centers
  - Gravitational forces acting at centers of mass

Analysis of Kinetics

- Assessment of total limb, multi-segment kinetics must be done for each segment sequentially
- Calculations start at the distal end of the limb, where external forces are defined
  - For leg - start at foot segment due to known ground reaction force
- Once proximal joint forces/moments are known, can proceed to next proximal segment
  - Distal forces/moments are equal and opposite to calculated proximal components

Analysis of Kinetics



- General equations for the foot:

$$\sum \vec{F} = m_f \vec{a}_f \qquad \sum \vec{M} = I_f \vec{\alpha}_f$$

- Simplified equations for the foot:

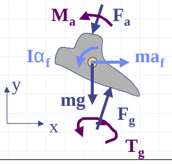
$$\vec{F}_a + \vec{F}_g + m_f \vec{g} = m_f \vec{a}_f$$

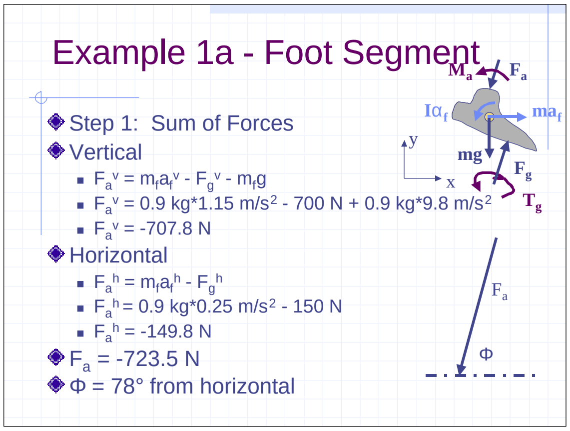
$$\vec{M}_a + \vec{T}_g + (\vec{r}_{cm,p} \times \vec{F}_a) + (\vec{r}_{cm,d} \times \vec{F}_g) = I_f \vec{\alpha}_f$$

- Similar equations can be written for the knee and hip

Example 1a - Foot Segment

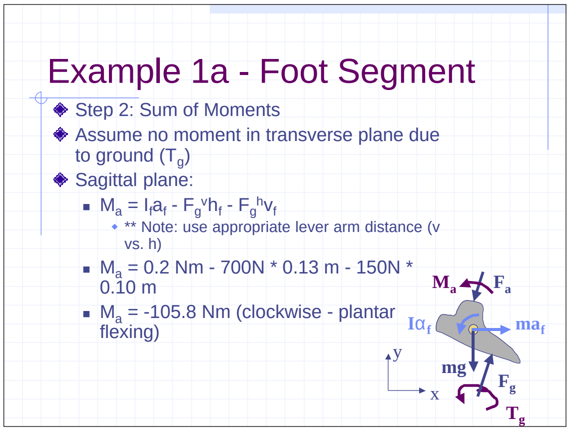
- Motion constrained to sagittal plane
- Knowns:
  - Ground reaction force: $F_g^v = 700$ N $F_g^h = 150$ N
  - Foot mass: $m_f = 0.9$ kg
  - Floor-ankle lever arm: $v_f = 0.10$ m $h_f = 0.13$ m
  - Foot acceleration: $a^v = 1.15$ m/s^2 $a^h = 0.25$ m/s^2
  - Foot inertia*ang accel: $I\alpha = 0.02$ Nm
- Unknowns:
  - Force at ankle
  - Moment at ankle





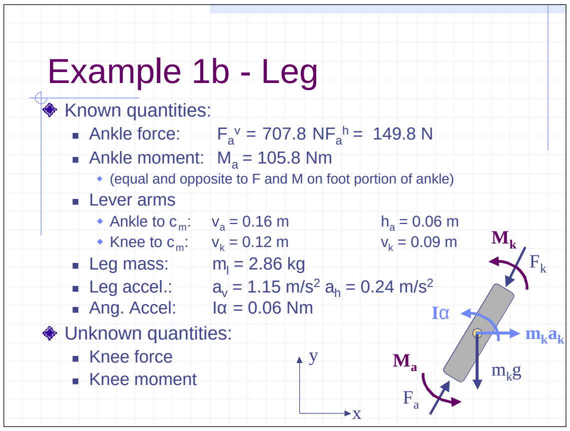
Example 1a - Foot Segment

- Step 1: Sum of Forces
- Vertical
  - $F_a^v = m_f a_f^v - F_g^v - m_f g$
  - $F_a^v = 0.9\ \text{kg} \cdot 1.15\ \text{m/s}^2 - 700\ \text{N} + 0.9\ \text{kg} \cdot 9.8\ \text{m/s}^2$
  - $F_a^v = -707.8\ \text{N}$
- Horizontal
  - $F_a^h = m_f a_f^h - F_g^h$
  - $F_a^h = 0.9\ \text{kg} \cdot 0.25\ \text{m/s}^2 - 150\ \text{N}$
  - $F_a^h = -149.8\ \text{N}$
- $F_a = -723.5\ \text{N}$
- $\Phi = 78°$ from horizontal



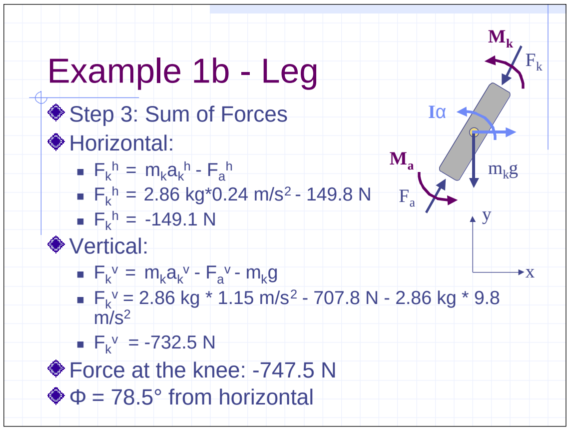
Example 1a - Foot Segment

- Step 2: Sum of Moments
- Assume no moment in transverse plane due to ground (T_g)
- Sagittal plane:
  - $M_a = I_f a_f - F_g^v h_f - F_g^h v_f$
    - ** Note: use appropriate lever arm distance (v vs. h)
  - $M_a = 0.2\ \text{Nm} - 700\text{N} \cdot 0.13\ \text{m} - 150\text{N} \cdot 0.10\ \text{m}$
  - $M_a = -105.8\ \text{Nm}$ (clockwise - plantar flexing)



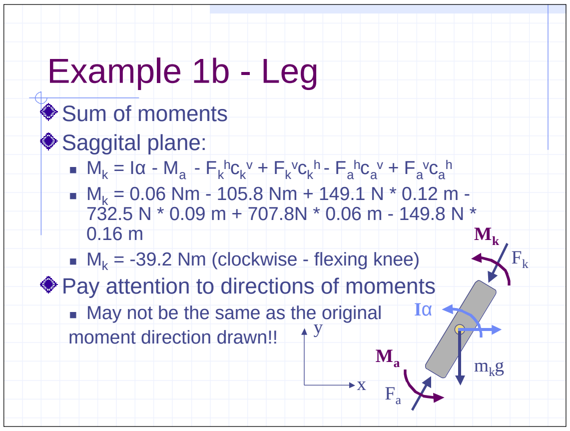
Example 1b - Leg

- Known quantities:
  - Ankle force: $F_a^v = 707.8\ \text{N}$$F_a^h = 149.8\ \text{N}$
  - Ankle moment: $M_a = 105.8\ \text{Nm}$
    - (equal and opposite to F and M on foot portion of ankle)
  - Lever arms
    - Ankle to c_m: $v_a = 0.16\ \text{m}$ $h_a = 0.06\ \text{m}$
    - Knee to c_m: $v_k = 0.12\ \text{m}$ $v_k = 0.09\ \text{m}$
  - Leg mass: $m_l = 2.86\ \text{kg}$
  - Leg accel.: $a_v = 1.15\ \text{m/s}^2$ $a_h = 0.24\ \text{m/s}^2$
  - Ang. Accel: $I\alpha = 0.06\ \text{Nm}$
- Unknown quantities:
  - Knee force
  - Knee moment



Example 1b - Leg

- Step 3: Sum of Forces
- Horizontal:
  - $F_k^h = m_k a_k^h - F_a^h$
  - F_k^h = 2.86 kg*0.24 m/s^2 - 149.8 N
  - F_k^h = -149.1 N
- Vertical:
  - $F_k^v = m_k a_k^v - F_a^v - m_k g$
  - F_k^v = 2.86 kg * 1.15 m/s^2 - 707.8 N - 2.86 kg * 9.8 m/s^2
  - F_k^v = -732.5 N
- Force at the knee: -747.5 N
- Φ = 78.5° from horizontal



Example 1b - Leg

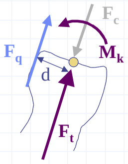
- Sum of moments
- Saggital plane:
  - $M_k = I\alpha - M_a - F_k^h c_k^v + F_k^v c_k^h - F_a^h c_a^v + F_a^v c_a^h$
  - M_k = 0.06 Nm - 105.8 Nm + 149.1 N * 0.12 m - 732.5 N * 0.09 m + 707.8N * 0.06 m - 149.8 N * 0.16 m
  - M_k = -39.2 Nm (clockwise - flexing knee)
- Pay attention to directions of moments
  - May not be the same as the original moment direction drawn!!

Equilibrium During Motion

- For joints to be in equilibrium during motion, the joint forces and moments must be balanced by internal forces
  - Muscle contractions
  - Ligament stretch
  - Articular reaction forces
- Equilibrium equations can be set-up to solve for six unknowns
- Generally more than six unknown internal forces
  - Indeterminant problem
- Reduction method can be used to create a determinant problem

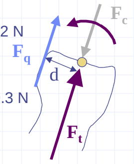
Example 2 - Internal Forces

- Known quantities:
  - Force through tibia: $F_t^h = 149.3$ N
    $F_t^v = 696.7$ N
  - Moment about knee: $M_k = 43.16$ Nm
  - Moment arm of quads: $d = 0.04$ m
- Unknown quantities:
  - Tibial-femoral contact force: F_c^v F_c^h
  - Quadriceps muscle force: F_q^v F_q^h
- Assumptions:
  - Hamstrings not active -- quadriceps act to extend joint
  - Angle of tibia: 60 degrees from horizontal



Example 2 - Internal Forces

- Equilibrium: $\Sigma F = 0$ $\Sigma M = 0$
- Moments:
  - $M_k - F_q{*}d = 0$
  - $43.2 \text{ Nm} - F_q{*}0.04 \text{ m} = 0$ $F_q = 1080$ N
  - $F_q^v = F_q \sin 60 = 935$ N $F_q^h = F_q \cos 60 = 540$ N
- Vertical:
  - $F_t^v + F_c^v + F_q^v = 0$
  - $696.7 \text{ N} + F_c^v + 935 \text{ N} = 0$ $F_c^v = 1632$ N
- Horizontal:
  - $F_k^h + F_c^h + F_q^h = 0$
  - $149.3 \text{ N} + F_c^h + 540 \text{ N} = 0$ $F_c^h = 689.3$ N
- Contact Force: $F_c = 1772$ N



Indeterminant Problems

- Reduction of an indeterminant problem can be accomplished
- Method 1
  - Muscle groups for knee motion:
    - Hamstrings - extension
    - Quadriceps - flexion
  - Ligament groups:
    - Cruciates (internal)
    - Collaterals (external)
  - No antagonist muscle activity

Indeterminant Problems

- Linear and non-linear optimization techniques can also be applied
- Estimates of maximum knee joint force range from:
  - 3.3 x BW (reduction method 1)
  - 7.1 x BW (optimization)
- Limitations in optimization techniques are due to:
  - Precision of kinematic data and determination of moment arms and external moments
  - Lack of knowledge of physiologic function and role of individual muscles during gait

Indeterminant Problems - Difficulties

- Examples of error sources
- Motion of knee joint changes the contact point of the joint with time
  - Changes moment arm of muscles
- Generated muscle force depends on muscle length and contraction velocity
- Physiological changes (fatigue)

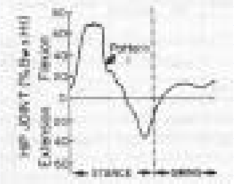
- Current techniques give us estimates of internal behavior

Joint Moments

- Joint moment can be assumed to be proportional to body weight x height
- Patterns exist in the change in joint moments through the gait cycle
- Flexion-Extension: between 1 and 3 patterns seen in population (hip, knee, and ankle)
- Abduction-Adduction: 1 pattern seen in population (hip and knee)
- Inversion-Eversion: 3 patterns seen in population (ankle)

Joint Moments - Hip

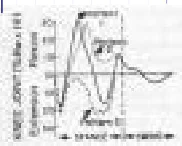
- Most reproducible flexion-extension moments between individuals
- Heel strike: moment flexes hip joint to maximum at mid-stance
- Mid stance: moment extends hip



From Mow and Hayes.

Joint Moments - Knee

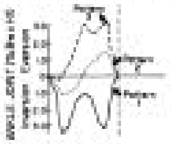
- Three major patterns of flexion-extension moment variation
- Pattern 1: 80% of population at normal speeds
  - Extension at heel strike
  - Flexion through midstance
  - Extension in late stance
  - Flexion prior to toe-off



- Pattern 2: 5% at normal speed
  - Flexes knee during entire mid to late stance
- Pattern 3: 15% at normal speed
  - Extends knee during entire mid to late stance
- Slow walking - more of patterns 2 and 3
- Fast walking - more of pattern 1

Joint Moments - Ankle

- Two patterns of flexion-extension moment transition
  - Plantarflexion at beginning of stance
  - Dorsiflexion throughout stance
    - Pattern 2 had slight fluctuation in rate of dorsiflexion
  - More population transitions to pattern 2 as pace of gait increases



- Three patterns of inversion-eversion moment
  - Foot may stay inverted (most common), stay everted, or transition from inversion to eversion

Joint Moments

- Magnitude of flexion-extension moments is dependent on walking speed
  - All joint f-e moments increase with walking speed
  - Exception is plantarflexion (remains constant)
- Internal forces are related to joint forces
- --> Internal forces can be expected to vary with walking speed
- Abduction-adduction and external-internal rotation moments do not vary with walking speed

Joint Moments

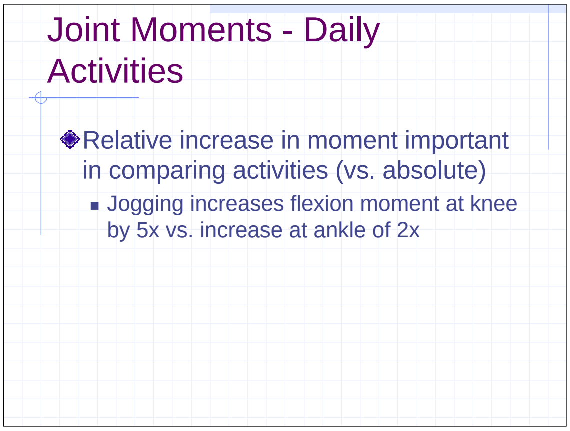
- The relation between normalized maximum joint moment and walking speed does not vary between men and women for flexion-extension
  - Differences in generated moment are based on variations in body weight and height
- Differences do exist between men and women in abduction-adduction moments
  - Hip moment 4% greater in women
  - Knee moment 8% greater in women
  - Assumed to be due to differences in pelvic structure

Joint Moments - Level Walking

- Largest moments occur during hip flexion and ankle dorsiflexion
  - More than twice maximum knee flexion moment
- Dorsiflexion moment is sustained throughout stance
  - Muscle endurance is required
- Phasic transitions of moments may act to reduce joint loading in normal gait
- Adduction moments are generally as high as extension moments

Joint Moments - Daily Activities

- Largest moments are generally during flexion of joints
- Hip:
  - Jogging > Climbing stairs > Rising from chair > Descending stairs > Walking
- Knee:
  - Jogging > Descending stairs > Rising from chair > Climbing stairs > Walking
  - Individuals with patellofemoral problems have problems descending stairs
- Ankle:
  - Jogging > Walking > Descending stairs > Climbing stairs > Rising from chair



Joint Moments - Daily Activities

- Relative increase in moment important in comparing activities (vs. absolute)
  - Jogging increases flexion moment at knee by 5x vs. increase at ankle of 2x