

**BME 5210**  
**Musculoskeletal Biomechanics**  
**Exam 1 – Winter 2001**

This is a closed book exam. Only the information provided with the exam may be used. Answer each question completely but concisely. You may use lists, phrases, or sentences.

By signing below you indicate that you have been informed about and are in compliance with the policy on academic dishonesty as it is applied to this class and this exam.

**Name:** \_\_\_\_\_

**Signature:** \_\_\_\_\_

1. Describe the structure of an osteon from its smallest constituent to the level of the whole structure. How does a primary osteon differ from a secondary osteon? (10 points)

*Hydroxyapatite (1) is imbedded in collagen fibers (1)*

*Collagen fibers arranged in parallel sheets (1)*

*Sheets of fibers are arranged in concentric layers (1) with alternating fiber directions (1)*

*The sheets of fibers are arranged around a vascular canal (1), with the axis parallel to the primary direction of loading (1)*

<u>Osteon Differences</u>	(1)	(1)	(1)
Primary	Smaller	No Cement Line	First osteons
Secondary	Larger	Cement Line	Form based on remodeling

2. Osteoporosis and osteopetrosis both lead to an increase in fracture risk. Describe how each disease acts to reduce the strength of the bones. (8 points)

*Osteoporosis: loss of bone (1), particularly of supporting trabecular struts (1). Reduces structural stiffness and strength (1). More prone to buckling failure (1).*

*Osteopetrosis: increased growth of cortical bone (1), but structure is disorganized (1). Structure does not provide optimum strength (1), will fail under lower load (1).*

3. Two basic hypotheses exist to describe the increased occurrence of fractures among the elderly. Describe both hypotheses. Which do you feel is more likely to explain this phenomenon and why? (8 points)

*Hypothesis 1: Decreased properties (volume fraction, thickness of bone, curvature) make structure more likely to fail under a given load. (2)*

*Hypothesis 2: Elderly individuals are more likely to have an adverse loading event, such as a fall. (2)*

*Your opinion, if substantiated, is worth up to four points.*

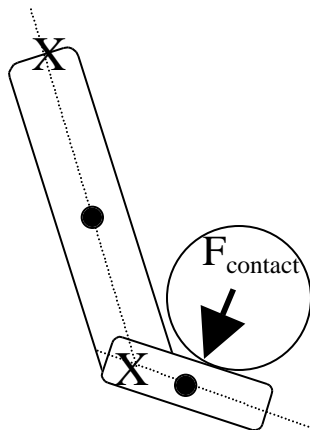
4. Explain why muscle forces in the body are generally much higher than the external forces they are trying to overcome when performing an activity. How does this affect the joint? (5 points)

*Muscles create a moment about a joint, which is needed to overcome the external moment caused by gravity or another externally applied force. The moment arm of the muscle is generally much shorter than that of the external force. Therefore, the muscle force must be greater than the external force in order to balance the moments.*

5. (25 points) A soccer player kicks the ball towards the goal with an acceleration of  $5 \text{ m/s}^2$ . The mass of the ball is  $0.5 \text{ kg}$ . Given the information below, what is the resultant joint force (magnitude and angle) and moment at the knee at the instant of contact between the ball and the foot. Assume the following:
- Planar motion (sagittal plane)
  - The center of mass of each segment is located halfway along its length and is on the same axis as the center of rotation of the joints
  - The foot is NOT in contact with the ground at the time of the kick

Parameter	Value
Mass of foot	0.9 kg
Mass of leg	4 kg
Length and thickness of foot	25 cm/4 cm
Length and thickness of leg	50 cm
Angle of foot	$-30^\circ$
Angle of leg	$110^\circ$
Acceleration of foot	$2 \text{ m/s}^2$ ( $30^\circ$ angle)
Acceleration of leg	$2 \text{ m/s}^2$ ( $0^\circ$ angle)
Angular acceleration of foot	$9 \text{ rad/s}^2$ (counterclockwise) about ankle
Angular acceleration of leg	$9 \text{ rad/s}^2$ (counterclockwise) about knee
Moment of inertia of foot	$0.0033 \text{ s}^2/\text{Nm}$ about ankle
Moment of inertia of leg	$0.015 \text{ s}^2/\text{Nm}$ about knee
Angle of contact force	$60^\circ$

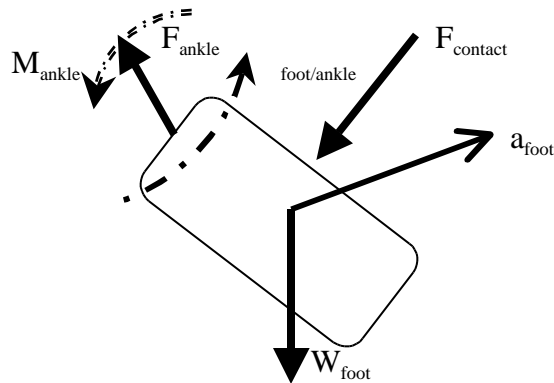
NOTE: All angle are given from the horizontal.



*Sum Forces = ma (1)*

*Sum Moments = I (1)*

Foot:



$$F_{\text{contact}} = F_c = m_{\text{ball}} a_{\text{ball}} = 0.5 \text{ kg} * 5 \text{ m/s}^2 = 2.5 \text{ N} (1)$$

*Vertical direction:*

$$m_j g + F_{av} + F_{cv} = m_j a_f (1)$$

$$-(0.9 \text{ kg})(9.8 \text{ m/s}^2) + F_{av} - 2.5 \text{ N}(\sin 60) = (0.9 \text{ kg}) (2 \text{ m/s}^2)(\sin 30) (1)$$

$$F_{av} = 11.8 \text{ N (directed upwards)} (1)$$

*Will exert -11.8 N (downward) force on leg (1)*

*Horizontal direction:*

$$F_{ah} + F_{ch} = m_j a_f$$

$$F_{ah} - 2.5 \text{ N}(\cos 60) = 0.9 \text{ kg} (2 \text{ m/s}^2)(\cos 30) (1)$$

$$F_{ah} = 2.81 \text{ N (directed to right)} (1)$$

*Will exert -2.81 N (left) force on leg (1)*

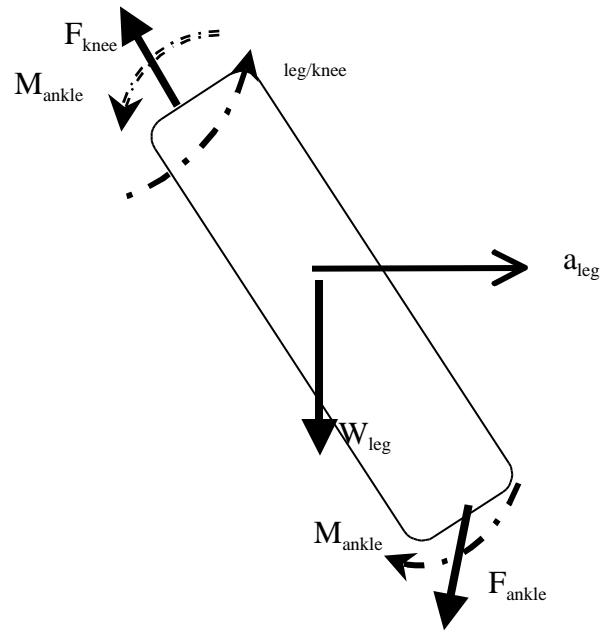
*Moments:*

$$M_a - F_c(12.5 \text{ cm}) - m_j g (12.5 \text{ cm} * \cos 30) = (0.0033 \text{ s}^2/\text{Nm})(9 \text{ rad/s}^2) (2)$$

$$M_a = 1.30 \text{ Nm} (1) \text{ (counterclockwise)}$$

*Will exert -1.30 Nm (clockwise) moment on leg (1)*

Leg:



Vertical:

$$\begin{aligned}F_{av} + m_l g + F_{kv} &= m_l a_{lv} = 0 \quad (1) \\-11.8 \text{ N} - 4 \text{ kg} (9.8 \text{ m/s}^2) + F_{kv} &= 0 \\F_{kv} &= 51 \text{ N} \quad (1)\end{aligned}$$

Horizontal:

$$\begin{aligned}F_{ah} + F_{kh} &= m_l a_{lh} \quad (1) \\-2.81 \text{ N} + F_{kh} &= 4 \text{ kg} (2 \text{ m/s}^2) \\F_{kh} &= 10.8 \text{ N} \quad (1)\end{aligned}$$

Resultant:

$$\begin{aligned}F_k &= (F_{kv}^2 + F_{kh}^2)^{1/2} \quad (1) \\F_k &= 52.1 \text{ N}\end{aligned}$$

$$\theta_k = \text{atan} (F_{kv}/F_{kh}) \quad (1)$$

$$\theta_k = 78^\circ \quad (1)$$

Moments:

$$\begin{aligned}M_a + M_k - m_l g (25 \text{ cm}) \cos 70 - F_{av} (50 \text{ cm}) \cos 70 - F_{ah} (50 \text{ cm}) \sin 70 \\= (0.015 \text{ s}^2/\text{Nm})(9 \text{ rad/s}^2) \quad (1) \\-1.3 \text{ Nm} + M_k - (4 \text{ kg})(9.8 \text{ m/s}^2)(25 \text{ cm}) \cos 70 - 11.8 \text{ N} (50 \text{ cm}) \cos 70 - \\2.81 \text{ N} (50 \text{ cm}) \sin 70 = (0.015 \text{ s}^2/\text{Nm})(9 \text{ rad/s}^2)\end{aligned}$$

$$M_k = 8.12 \text{ Nm counterclockwise} \quad (1)$$

6. (10 points) When a ballerina dances on her toes, her foot is in the fully plantar flexed position. For a body mass of 50 kg, what is the Achilles tendon force needed to maintain the foot in this position when in a single legged stance (such as a pirouette). The following data is available:

Distance from Achilles tendon to center of rotation: 2 cm  
 Distance from line of action of ground reaction force to center of rotation of ankle: 3 cm  
 Achilles tendon acts in a purely vertical direction

If the dancer stumbles and her center of gravity shifts forwards by 10 cm, what tendon force would be needed to maintain equilibrium with the foot plantarflexed?

(NOTE: The Achilles tendon transfers the force from the gastrocnemius in the posterior portion of the leg to the foot.)

$$F_{gr} = BW = 50 \text{ kg} (9.8 \text{ m/s}^2) = 490 \text{ N} (2)$$

$$\text{Sum of forces} = 0 (1) \quad \text{Sum of moments} = 0 (1)$$

$$a) F_{gr} (3 \text{ cm}) - F_{at} (2 \text{ cm}) = 0 (1)$$

$$490 \text{ N} (3 \text{ cm}) - F_{at} (2 \text{ cm}) = 0$$

$$F_{at} = 735 \text{ N} (2)$$

b) Option 1:

Assume ground reaction force is in line with center of gravity

$$F_{gr} (13 \text{ cm}) - F_{at} (2 \text{ cm}) = 0 (1)$$

$$490 \text{ N} (13 \text{ cm}) - F_{at} (2 \text{ cm}) = 0$$

$$F_{at} = 3185 \text{ N} (2)$$

Option 2:

Recalculate  $F_{gr}$  based on off set center of gravity

$$F_{gr} - BW - F_{ankle} = 0$$

$$F_{gr} = 490 \text{ N} + F_{ankle}$$

$$- BW (13 \text{ cm}) + F_{gr} (3 \text{ cm}) = 0 \text{ (balance moment about ankle)}$$

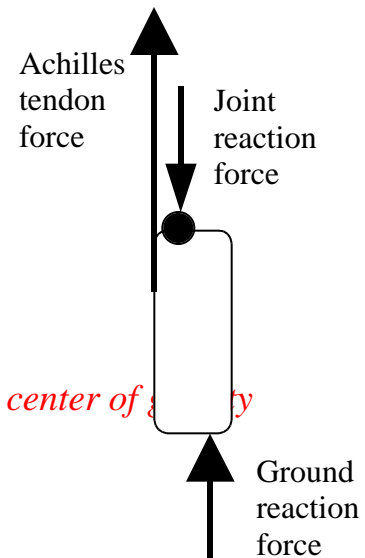
$$F_{gr} = 490 \text{ N} (13 \text{ cm}) / (3 \text{ cm}) = 2123 \text{ N}$$

Balance internal moments:

$$F_{gr} (3 \text{ cm}) - F_{at} (2 \text{ cm}) = 0 (1)$$

$$(2123 \text{ N}) (3 \text{ cm}) - F_{at} (2 \text{ cm}) = 0$$

$$F_{gr} = 3185 \text{ N}$$



7. The gastrocnemius is not the only muscle that is involved with plantarflexion of the foot. Describe in detail one way in which you could solve the indeterminate problem that would exist if you were to consider the role of the gastrocnemius and the soleus muscles. (10 points)

Reduction:

*Assume contribution from both gastrocnemius and soleus*

$$F_{TOT} = F_{GN} + F_{SOL}$$

*The distribution of the muscle force can be assumed to be dependent on the PCSA*

$$F_{max} = \sigma_{max} PCSA$$

*Using this assumption, the proportion of force in each of the muscles can be estimated*

$$F_{GN} = (PCSA_{GN}/PCSA_{TOT})F_{TOT} \text{ etc.}$$

*The number of equations is therefore reduced by one, making the problem determinate*

*NOTE: Grouping the muscles together will not allow you to consider the role of both the gastrocnemius and the soleus – you would simply be considering the role of plantarflexors in general, and you would get the same solution you had in problem 6!*

Optimization:

*Set up the equilibrium equations based on known parameters (geometry, muscles involved, etc.)*

*Determine your optimization parameter and function (ie. Minimize total force, minimize energy, etc.)*

*Solve the problem using a computer algorithm to optimize the selected function*